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STUDIES OF NONLINEAR PHENOMENA IN HIGH - TEMPERATURE PLASMAS

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PRINCIPAL INVESTIGATOR:
P.K.C. WANG

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This report summarizes the results of a study on various nonlinear phenomena in high-temperature plasmas; in particular, explosive instabilities and wave-energy transfer in nonlinear plasmas, turbulence, numerical solution of nonlinear equations in plasma physics, plasma heating by neutral injection, feedback stabilization and control of high-temperature plasmas.

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FINAL TECHNICAL REPORT

for

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PRINCIPAL INVESTIGATOR:

P.K.C. WANG

School of Engineering and Applied Science
University of California
Los Angeles, California 90024

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ABSTRACT

This report summarizes the results of a study on various nonlinear phenomena in high-temperature plasmas; in particular, explosive instabilities and wave-energy transfer in nonlinear plasmas, turbulence, numerical solution of nonlinear equations in plasma physics, plasma heating by neutral injection, feedback stabilization and control of high-temperature plasmas.

1. INTRODUCTION

Recent experiments in plasma physics involving the interaction of high-intensity electromagnetic radiation with plasmas, heating of plasmas by injecting high-energy particles, and high-current toroidal plasmas often produced results which cannot be explained using linear theory. A typical example is the explosive instabilities resulting from the interaction of a system of positive and negative energy waves in a plasma. The induced emission of high-intensity electromagnetic radiation from a high-temperature plasma is not predictable by means of linear models. Also, the onset of turbulence in a plasma is basically a nonlinear phenomena. It is evident that nonlinear phenomena will play an increasingly important role in the future development of both theoretical and applied plasma physics.

From January 1,1974 through December 31,1978, studies of nonlinear phenomena in high-temperature plasmas under the support of AFOSR grant No. 74-2662 have been concentrated in the following three main areas: (a) nonlinear plasma theory, (b) plasma heating by neutral-particle injection and (c) feedback stabilization and control of plasmas. The results in each of these areas will be summarized categorically below.

2. NONLINEAR PLASMA THEORY

Studies in nonlinear plasmas began with an investigation of nonlinear wave-wave interactions in plasmas with particular emphasis on explosive instabilities. This work was motivated from a number of experiments on the wave-heating of plasmas in which wave amplitudes with nonexponential growth rates were observed. Later, this work was broadened to include plasma turbulence theory from the viewpoint of bifurcation theory and soli-

ton interactions. In parallel with this work, attempts were made in developing efficient accurate numerical methods for solving the nonlinear plasma equations.

2.1 Explosive Instabilities and Wave-Energy Transfer in Nonlinear Plasmas

It is well-known that in nondissipative plasmas with nonlinearly interacting positive and negative energy waves, the wave amplitudes could grow at rates faster than exponential [1]-[3]. Moreover, some of the waves may exhibit an explosive behavior in the sense that their wave amplitudes tend to infinity in a finite amount of time. Of course, in physical situations, various saturation phenomena take place when the wave amplitude becomes sufficiently large. In plasmas, explosive instabilities have been observed in a number of experiments [4],[5]. During recent years, theoretical studies in this area have been focused on the derivation of necessary and/or sufficient conditions for the existence or nonexistence of explosive instabilities in plasmas with nonlinearly interacting waves. For the case of nondissipative nonlinear plasmas having waves with deterministic phases, conditions for the existence of explosive instabilities have been established for the case with weakly nonlinear wave-wave interactions [6]. Wilhelmsson et al [7] derived a necessary condition for explosive instability for three-wave interactions taking into account the effect of linear damping or growth. In a study supported by AFOSR grant 72-2303, Wang [8] obtained a sufficient condition for the nonexistence of explosive instabilities by first deriving appropriate bounds for the solutions of the differential equations governing the dynamics of nonlinear wave-wave interactions. In spite of the large amount of existing work pertaining to the explosive instabilities in various types of nonlinear plasmas, no attempt has been made to unify the existing results or to approach the problem from a general viewpoint. Most

works dealt with very special models of nonlinear plasmas and classical mathematical techniques were used in the analysis.

In this study, we tried to approach the problem from the standpoint of qualitative theory of differential equations. The objective is to obtain some general results pertaining to the existence and nonexistence of explosive solutions of those classes of differential equations arising in nonlinear plasma theory. Then, we proceeded to investigate particular mathematical models of nonlinear plasmas in the light of the general results.

The starting point of this study is based on the observation that the differential equations describing almost all the nonlinear plasmas can be formulated in the form of a nonlinear evolution equation of the form:

$$\frac{dx}{dt} = Ax + f(x), \tag{1}$$

defined on some suitable Hilbert space H, where A is a linear operator on H into H, and f is a nonlinear operator having the homogeneity property $f(cx) = c^k f(x)$ for some $k > 0$. For example, in the case of a system of a finite number of nonlinearly interacting waves in a plasma, equation (1) corresponds to a system of ordinary differential equations defined on the n-dimensional complex vector space C^n with x being the complex wave-amplitude vector. On the other hand, for a nonlinear plasma described by the nonlinear fluid equations, (1) corresponds to a system of partial differential equations in which A is a linear spatial differential operator and f is a nonlinear homogeneous spatial differential operator with degree of homogeneity $k = 2$.

In this study, attention was focused on developing general sufficient

conditions for the existence and nonexistence of explosive solutions of (1). Results were obtained by projecting the solutions of (1) onto the unit sphere in the space H and then studying the behavior of the solutions which lie along rays emanating from the origin of the space. It was shown that explicit conditions for which the ray solutions are explosive or non-explosive can be obtained by solving an eigenvalue problem associated with the operators A and f. When A and f belong to certain special classes of operators, in particular, potential operators and compact operators, the existence of explosive solutions can be established directly in terms of the parameters of (1). Moreover, the explosion times for the solutions can be computed exactly. The aforementioned classes of operators arise in nonlinear wave-wave interactions in plasmas. The details of these results are described in paper (P-2).

In order to determine the implications of the general theoretical results, studies were made on the nonlinear interaction of a finite number of waves in plasmas. It was shown that the wave energy transfer along any ray solution is always unidirectional or irreversible, i.e. a wave gains or loses energy along a ray solution at all times. This property is significant from the application standpoint, since it implies that the ray solutions represent efficient paths for transferring energy from one wave to another. Evidently, this type of solutions may be selected for heating a plasma to high temperatures (for example, by transferring energy from an intense external wave to the plasma waves). Detailed results were worked out for the special case of nonlinear two-wave and three-wave interactions. The significance of these results for the nonlinear resonant interaction of three longitudinal electromagnetic waves in a magnetized collisionless plasma was also studied. The results of this aspect

of study were published in paper (P-5). Pursuing further along these directions, it was found during the first part of 1975 that the unidirectional energy transfer property not only holds along the ray solutions of a general wave-wave interacting system, but also along those solutions whose phases are invariant with time. This suggests that in the wave-heating of plasmas, the initial phases of the waves could be chosen in such a way to achieve more efficient transfer of energy. The results of this study were first presented at the Second International Congress on Waves and Instabilities in Plasmas held in Innsbruck (Paper (CP-1)) and published later in Physica (Paper (P-7)). Also, the results pertaining to the wave energy transfer in time-varying nonlinear wave-wave interacting systems were obtained and published in paper (P-1).

In late 1975, an attempt was made to derive more general sufficient conditions for the existence or nonexistence of explosive instabilities. In early 1976, we obtained a set of general conditions for stability and for explosive instability by using certain relations between an abstract function and its Gateaux differential. It was shown that these results are applicable to a large class of equations arising from physical systems involving nonlinear wave propagation, diffusion or transport delays. A special case of these results is the well-known Krasovskii's stability theorem for finite-dimensional ordinary differential equations. These results were published in the paper (P-8).

In the study of explosive instabilities and wave-energy transfer in specific wave-wave interacting systems, results were obtained for the parametric decay of an intense, coherent electromagnetic wave into a plasma wave and scattered electromagnetic waves in a homogeneous plasma (stimulated Raman scattering). This study includes the coupling of secondary waves

consisting of a plasma wave harmonic and its associated mixed electromagnetic-electrostatic sidebands. Explicit expressions were obtained for the wave amplitudes and nonlinear growth rates when $t > 1/\gamma$, where γ is the linear growth rate of the fundamental. It was shown that, in this regime, the nonlinear growth rate is $2\gamma/3$. The results are described in report (R-2).

Finally, a general formalism was developed to describe the nonlinear evolution associated with the parametric decay of an intense, coherent electromagnetic wave into an electrostatic wave, its second harmonic, and scattered electromagnetic waves in a homogeneous plasma. Neglecting the effects of pump depletion and assuming all waves are coherent, two classes of solutions were found. One class consists of explosively unstable solutions and the other consists of growing aperiodic oscillatory solutions. The details are given in report (R-3). These results have applications in ionospheric modification experiments and in the interaction of laser beams with plasmas.

2.2 Plasma Turbulence

A possible approach to developing a theory for strongly turbulent plasmas is via the study of the interaction of solitons. In 1973, Kaplan and Tsytovich [7] showed by means of quasi-linear theory that the energy in a Langmuir-turbulent plasma tends to accumulate at very small wave numbers. However, in view of the results of Vedenov and Rudako [8], it is apparent that this nearly uniform plasma state is unstable. In fact, modulational instability can occur which leads to local density depressions along with increases in the energy density of the Langmuir waves. In 1972, Zakharov [9] formulated the basic equations governing the modulational instability in which the high-frequency Langmuir oscillations are coupled

to the ion motion by the ponderomotive force. It was found, in the one-dimensional case, that these equations possess soliton solutions in which the Langmuir oscillations are trapped in regions of lower density. Later, Kingsep et al [10] suggested that strong Langmuir turbulence could be described in terms of a system of interacting Langmuir solitons. A considerable amount of work both analytical and numerical, has been done based on this idea for the one-dimensional case [11]-[13]. It was hoped that similar theory could be developed for the three-dimensional case. During 1977, we obtained general sufficient conditions for the existence or nonexistence of a class of multi-dimensional Langmuir solitons based on a generalized version of the Zakharov's model. It was shown that these solitons having properties similar to those of the one-dimensional case could exist under subsonic conditions. Also, periodic nonlinear travelling waves could form under both subsonic and supersonic conditions. The results were applied to the special cases with the usual ponderomotive force and with ion density saturation. The results were published in paper (P-9).

In early 1978, we began exploring a new direction for developing a deterministic theory for plasma turbulence. This work was motivated from Ruelle-Takens' proposal [14] on the mathematical characterization of fluid turbulence in terms of "strange-attractor solutions". This class of solutions corresponds to those trajectories in the state space of the dynamical model which are attracted to a nonempty set ("strange attractor"). This set is neither an equilibrium set nor a periodic orbit. Moreover, the autocorrelation function $\rho(\tau)$ for these solutions tends to zero as the separation time $\tau \to \infty$. In fluid turbulence, Lorenz's simplified model for thermal convection in the atmosphere seems to have solutions exhibiting such

properties [15]. In view of the similarities of the differential equations in the Lorenz model and those describing nonlinear wave-wave interactions in plasmas, one might expect to find similar solutions for plasma models. In late 1978, it was found that the single-mode equations derived from the Zakharov's model for Langmuir turbulence in a plasma in the presence of an externally spatially homogeneous electric field oscillating at the electron plasma frequency indeed has strange-attractor-like nonperiodic solutions. Moreover, their power spectra have turbulence-like features. Numerical studies were made for the one-dimensional case. These results suggest that, in the case of multiple modes, if all the mode coupling terms are omitted, each mode is capable of producing turbulence-like solutions independently. This seems to imply that energy transfer between various modes is not necessary in producing turbulence, which is contrary to the classical cascade theory of turbulence. These results, or course, are preliminary in nature. However, they represent a new direction of study for plasma turbulence which is based on deterministic rather than stochastic models. The details of this work were described in the Report (R-4).

2.3 Numerical Solution of Nonlinear Equations in Plasma Physics

In the study of nonlinear models for plasmas, it is often necessary to ckeck the range of validity of the qualitative theoretical results through a numerical study of the model equations. Although much work has been done on the numerical solution of the plasma equations with self-consistent fields, most of the existing works are based on finite-difference methods which generally lead to inefficient computational algorithms.

In this study, special emphasis has been focused on the application of the finite-element method for solving the nonlinear plasma kinetic equations. This method has been successfully used in solving the equations of nonlinear

elasticity. It has the capability of achieving high accuracy with a small number of approximating equations. Here, the numerical solution of the Vlasov-Poisson system of nonlinear equations for a multispecies plasma by means of the finite-element Galerkin approximation was considered. The following numerical algorithm was proposed: First, the time interval is partitioned into subintervals. At the beginning of each subinterval, the Poisson's equation with the charge density at the end of the previous subinterval is solved. Then, the linearized Vlasov's equations are solved using the finite-element Galerkin approximation. Explicit error estimates were derived. Using these estimates, it is shown that the approximate solutions generated by the proposed algorithm converge to the true solution. The rate of convergence was also established. To gain some computational experience with the proposed algorithm, a computer code was developed for the case of an electron-ion collisionless plasma. The numerical results compared favorably with the theoretical error estimates. The details are presented in the Ph.D. thesis by Ezzuddin (T-1).

A study was also made on the numerical solution of the equations for non-equilibrium nozzle flow in CO_2-N_2 gasdynamic lasers. A computational method based on optimal control theory was developed. In this method, the nozzle area and the gradient of the gas temperature are treated as the state and control variables respectively, and the non-equilibrium effects due to the kinetic processes are considered in both the upstream and downstream of the nozzle throat. This approach bypasses the difficulties due to the sonic-point singularity. The results of this study are given in the M.S. thesis by Masui (T-2).

3. PLASMA HEATING BY NEUTRAL INJECTION

During the first half of 1975, the principal investigator visited the Department of Plasma Physics and Controlled Fusion, Center of Nuclear Studies at Fontenay-aux-Roses, France. A study in collaboration with M. Cotsaftis was made on the optimal heating of plasmas by neutral injection. A mathematical model for the time evolution of the spatially averaged electron and ion temperatures was used in this study. The model takes into account the major sources and sinks which are important in the KeV range. First, the dynamics of plasma heating was analyzed in the electron-ion temperature plane for the case with constant energy and current neutral injection. By investigating the behavior of the right-hand-sides of the temperature equations, the exact regions of heating and cooling of electrons or ions were found. It was shown that there exists a region in which heating of both the electrons and ions takes place, and the size of this region increases with the injection current or energy. Figure 1 shows the qualitative behavior of typical trajectories in the electron-ion temperature plane for the case with constant current and energy injection.

The second phase of this study pertains to various problems associated with the optimization of the neutral injection parameters. In particular, the problem of minimizing the total input energy to achieve the highest ion temperatures at the end of a specified injection period was studied. The problem of time-optimal heating was also investigated. Here, an injection current and/or energy program was sought to attain a given value of ion-temperature in a minimum amount of time subject to certain limitations on the maximum values of the injection current and energy. These problems were studied in the spirit of optimal control theory. The results of this study were applied to the Tokamak device at Fontenay-aux-Roses. Good agreement

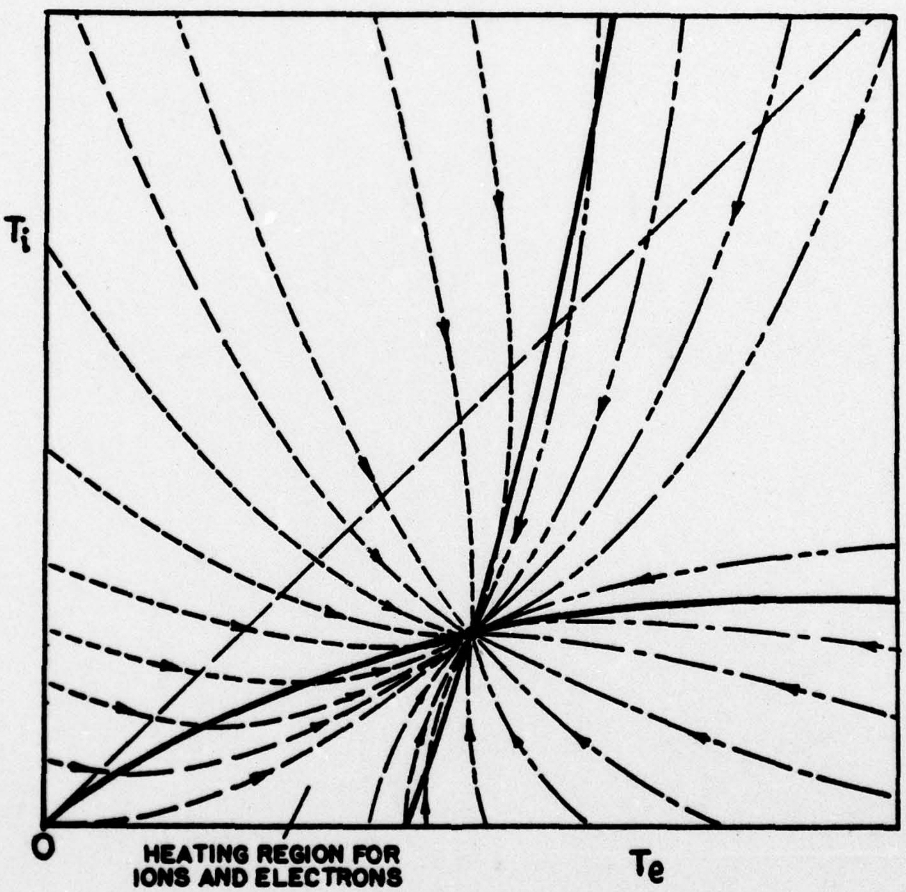

Figure 1. Typical electron and ion temperature trajectories with constant injection energy and current.

was obtained between the theoretical and experimental results. These results were presented at the American Physical Society, Plasma Physics Division Meeting in St. Petersburg, Florida, in 1975 (Conference Papers CPA-3 and CPA-4). They are also described in the report (R-1).

This study was continued during M. Cotsaftis' visit to U.C.L.A. in 1976. The main idea of the continuing study was motivated from the results of the neutral injection experiments at Fontenay-aux-Roses during 1976. It was observed that the electron temperature did not vary appreciably during the neutral injection period while the ion temperature increased substantially. This could be attributed to the fact that, when the plasma current was sufficiently large, the electrons were in a high-loss regime dominated by trapped-electron instability. In this case, the two-temperature model developed earlier could be simplified by setting the electron temperature at its equilibrium value. Based on this simplified model, we solved completely the problem of finding an injection current program $I = I(t)$ satisfying an amplitude constraint such that the ions are heated to a specified temperature while minimizing the heating time or the injection energy over a given time interval. The dynamics and stability of an ion-temperature feedback control system were also analyzed for various forms of nonlinear feedback controls. The analysis included the effect of measurement time-delay on system stability. The results suggest the possibility of using automatic control for close regulation of the ion-temperature by means of neutral injection. The problem of identifying the ion diffusion law by measuring the ion temperature over a finite period of time was also explored. The results of this study are described in paper (P-10) and were presented at the 1976 Annual Meeting of the Plasma Physics Division of the American Physical Society in San Francisco.

Finally, the problem of optimal timing and current programming of neutral beam injection to achieve the most efficient heating of plasmas was investigated. It was shown that this problem can be formulated as a multi-stage optimal control problem. First, certain general results pertaining to the existence and characterization of optimal controls were obtained. Then, they were applied to the optimal neutral injection heating problem. The results are presented in the Ph.D. dissertation by K. Tomiyama (T-3).

4. FEEDBACK STABILIZATION AND CONTROL OF HIGH-TEMPERATURE PLASMAS

In the production of high-temperature laboratory plasmas, it is desirable to be able to vary or regulate various plasma parameters such as temperature, density, etc., as required by a particular experiment. Due to the inherent instabilities and physical constraints, the ranges of allowable parameter variations are narrow. Consequently, such plasmas cannot be used for a wide range of experiments. By introducing feedback controls to regulate the plasma parameters, it may be possible to extend the useful parameter range, thus enhancing the usefulness of the laboratory plasma.

The main objective of this study is to develop a theory for the feedback stabilization and control of high-temperature plasmas from a unified control theoretic viewpoint. The specific areas of this study will be discussed separately below.

4.1 Feedback Stabilization

Given a mathematical model for a plasma to be stabilized by means of some form of external controls (for example, manipulatable currents, electric fields, etc.), the following basic questions should be answered before attempting to implement any feedback controls: (1) Do the controls affect all the

unstable motions of the plasma? (2) Where should the controls and sensors be placed to attain the most effective stabilization? (3) What is the set of all stabilizing feedback controls for the given mathematical model of the plasma? (4) Does there exist a feedback control which is realizable by using meaurable plasma parameters only?

In this study, satisfactory answers to the foregoing questions have been obtained for highly conducting plasmas whose motions about an unstable equilibrium are describable by the linearized MHD equations. The controls correspond to external currents imbedded in a vacuum region surrounding the plasma. It was shown that certain internal modes and the interchange modes cannot be controlled if the control is effective over the entire plasma surface. Complete control can be achieved, however, if we restrict the control region to a portion of the plasma surface. This implies shielding of the control fields. It was also shown that, for a plasma with a finite number of unstable modes, complete stabilization can be achieved by feeding back a suitable linear combination of the unstable model amplitudes only and by choosing the control region properly. The results are described in (C-1).

A particular form of feedback controls which is especially useful in laboratory plasmas is the so-called noninteracting or decoupled controls. The basic idea here is to introduce independent controls equal in number to that of the variables or parameters to be controlled. In general, each actuating control may affect more than one controlled variable, or each controlled variable may be affected by more than one actuating control. For example, in a laboratory plasma with a finite number of unstable modes to be controlled, the actuating controls may be in the form of probes, external current or particle sources. Each of them may affect more than one unstable

mode. It is desirable to have a control system such that each control (input command) affects only one mode and each mode is affected by only one control. Thus, there is a one-to-one correspondence between the input command and the modes to be controlled. This may be accomplished by introducing suitable feedback signals within the plasma system. During the period covered by this report, a study was made on the noninteracting controls for high-temperature plasmas. Attention was focused on developing theories for noninteracting modal controls for plasmas describable by the MHD equations and the Vlasov equation. Both the MHD modes and drift modes were considered. Results were obtained pertaining to the following basic questions: (1) Given a mathematical model for the plasma, under what conditions can one achieve the noninteracting controls? (2) What should be the form and the location of the controls and sensors for implementing the noninteracting controls? The details are given in (C-1) and were presented at the First IEEE International Conference on Plasma Science (see Conference paper CPA-1).

The practical significance of these results is that, once the noninteracting modal controls have been achieved, the problem of stabilization of unstable modes becomes straightforward. Moreover, each mode can be stabilized or enhanced separately by introducing feedback of the modal amplitude of that mode only, without affecting the remaining modes. This feature is very useful from an experimentalist's viewpoint, since each mode can be completely controlled and studied in detail at the experimentalist's discretion. In plasma experiments, the noninteraction controls are also useful in achieving <u>independent</u> adjustment of the important plasma parameters.

4.2 Optimal Control

During the period covered by this report, various new optimal control problems motivated by plasma control were studied. In particular, the problem of optimal control of systems governed by parabolic differential equations with boundary conditions involving time-delays was investigated. This problem was derived from that of confining a collision-dominated plasma by means of reflective barriers (such as a magnetic mirror field) at the plasma-domain boundary. The details are published in paper (P-3). In the same vein, optimal control problems for time-lag systems with time-lag controls were studied (see papers (P-4) and (P6)). Finally, it was shown that the problem of optimal confinement of plasmas by means of external electromagnetic fields can be formulated mathematically as a problem in the optimization of set functions; i.e., by choosing a geometric domain in a certain class to minimize a given functional. General necessary and sufficient conditions for optimality were obtained for this class of problems (see (P-11), (CP-3) and (T-4) for details).

4.3 Parameter Identification in High-Temperature Plasmas

In the generation of high-temperature plasmas, it is of importance to estimate the basic plasma parameters, such as density and temperature, from the physically observable quantities. In the implementation of feedback controls for plasmas, it is also necessary to have good estimates of the important plasma parameters, which are usually in the nonequilibrium state. There are two basic aspects to this problem, namely, devising suitable measurement or diagnostic techniques for plasmas, and processing the experimental data to obtain the best estimates of the plasma parameters. Existing work in plasma diagnostics deals primarily with developing new measurement tech-

niques. On the other hand, in the area of system identification, statistical estimation techniques have been used to process the measurement data. A unification of the techniques in plasma diagnostics and system identification could lead to significant improvement in the accuracy of estimation of the plasma parameters from the experimental data.

Here, studies were made on various parameter estimation problems in plasma physics from the viewpoint of system identification. Particular attention was focused on plasma parameter estimation based on electromagnetic-wave scattering data. Both coherent and incoherent scattering were considered. The parameter estimation problems were formulated on the basis of a least-squares criterion. The problem of identifying the electron diffusion law in toroidal plasmas from plasma current and electron temperature measurements was also considered. The computational aspects of parameter estimation were also explored using gradient type algorithms in processing the experimental data. The details are given in (P-10), (CP-2) and (CPA-5). An important advantage of the foregoing approach is that the parameters associated with nonequilibrium plasmas or time-varying phenomena can be estimated.

5. PUBLICATIONS, CONFERENCE PAPERS, THESES, REPORTS, LECTURES AND SEMINARS

Journal Papers

(P-1) P.K.C. Wang, "Wave Energy Transfer in Time-Dependent Nonlinear Wave-Wave Interactions," *Il Nuovo Cimento, Vol. 24B* (1974) 63-77.

(P-2) P.K.C. Wang, "Ray and Explosive Solutions of Nonlinear Evolutional Equations in Hilbert Spaces," *J.Math.Physics, Vol.15, No.2* (Feb 1975) 251-256.

(P-3) P.K.C. Wang, "Optimal Control of Parabolic Systems with Boundary Conditions Involving Time Delays," *SIAM.J.Control, Vol. 13,No.2* (Feb 1975) 274-293.

(P-4) P.K.C. Wang, "Optimal Control of Discrete-Time Systems with Time-Lag Controls," *IEEE Trans. on Auto.Control, Vol.AC-20, No.3* (June 1975) 425-425.

(P-5) P.K.C. Wang, "Explosive Ray Solutions of Nonlinear Wave-Wave Interacting Systems," *Il Nuovo Cimento, Vol.28b, No.1* (1975) 56-68.

(P-6) P.K.C. Wang, "Time-Optimal Control of Time-Lag Systems with Time-Lag Controls," *J.Math.Anal.Applications, Vol.52, No.3* (Dec.1975) 366-378.

(P-7) P.K.C. Wang, "Phase-Invariant Solutions of Nonlinear Wave-Wave Interacting Systems," *Physica, Vol. 814* (1975) 441-453.

(P-8) P.K.C. Wang, "Stability and Instability Conditions for Nonlinear Evolutional Equations in Hilbert Spaces," *J.Math.Physics, Vol.17, No.8* (1976) 1414-1420.

(P-9) P.K.C. Wang, "A Class of Multidimensional Nonlinear Langmuir Waves," *J.Math.Physics, Vol.19* (1978) 1286-1292.

(P-10) M. Cotsaftis, K. Tomiyama and P.K.C. Wang, "Ion Temperature Control in Toroidal Plasmas by Neutral Injection," *Intl.J.Control*, (to appear in 1979).

(P-11) R. Morris, "Optimal Constrained Selection of a Measurable Subset," *J.Math.Analysis and Applications* (to appear in 1979).

Chapter in Book

(C-1) P.K.C. Wang and G. Rodriguez, "Feedback Stabilization of Hydromagnetic Equilibria," in *Identification, Estimation and Control of*

Raman scattering). This study includes the coupling of secondary waves

5

Distributed-Parameter Systems, (edited by W.H. Ray and D.G. Lainiotis) Marcel Dekker, NY, 1978 (Chapter 8, pp.457-495).

<u>Conference Papers</u>

(CP-1) P.K.C. Wang, "Phase-Invariant and Ray Solutions of Nonlinear Wave-Wave Interactions in Plasmas," *Proc. 2nd Intl. Congress in Waves and Instabilities in Plasmas, Mar. 1975, Innsbruck, Austria*, Paper B-11.

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